Shock Capturing Using a Pressure-Correction Method

James J. McGuirk* and Gary J. Page†
Imperial College of Science, Technology and Medicine, London, England, United Kingdom

A new pressure-correction scheme has been developed, which is suitable for the calculation of a flow containing a wide range of Mach numbers such as a transonic impinging jet. The method uses equations based on properties per unit volume so that momentum is retained as a basic dependent variable rather than velocity. This simplifies the discretization of the time-dependent flow equations and allows a direct relationship to be determined between pressure and mass flux. The hyperbolic nature of the system of equations is obtained by using the retarded pressure approach. This is a transformation of the real pressure based on local Mach number and is used in the momentum and pressure-correction equations. The shocked quasi-one-dimensional flow in a nozzle is used as a test of shock capturing properties and speed of computation. The new method gives precise shock capturing over two nodes with no over or under-shoots; it is also significantly faster than the MacCormack and Jameson explicit schemes tested for this problem. Finally, a turbulent, under-expanded, axisymmetric, impinging jet calculation is presented. The correct periodic under/over-expansions of the jet are predicted, and the normal standoff shock is cleanly captured.

Introduction

AN Advanced Short Take-Off and Vertical Landing (ASTOVL) fighter aircraft may require the use of high pressure ratio, supersonic jets for propulsion and lift. As a consequence, attention must be paid to ground-effect problems arising in the flowfield beneath such an aircraft during landing with downward vectored impinging jets. Computational fluid dynamics will be a valuable tool in these studies. Calculations will be required in flow domains containing supersonic expansions, compressions, and strong normal stand-off shocks in the jet cores and with significant subsonic regions at very low Mach numbers in which turbulent mixing is important. This combination of flow regimes within the same solution domain provides a daunting test of any flow algorithm. A suitable computational method must be able to adequately model both compressible phenomena such as shock waves as well as almost incompressible turbulent features of the flow.

Most current compressible flow techniques have their origins in external aerodynamic problems and retain density as the subject of the continuity equation. Examples of such techniques are the widely used Jameson and MacCormack methods. Density is treated as a transported variable; pressure is then derived from an equation of state. However, the accuracy and efficiency of these practices at very low Mach numbers is questionable. Such methods do give reasonable shock capturing, although artificial viscosity is necessary to suppress oscillations. To date, however, they have rarely been used for the type of turbulent, recirculating flows typical of jet impingement problems.

Pressure correction methods, on the other hand, are effectively characterized by the extraction of an equation for pressure from the continuity equation; any change in density is then considered as a function of pressure via an equation of state. These are in widespread use for complex, recirculating, turbulent flows, although extensions to compressible (and certainly transonic) regimes are few and inadequately tested—particularly in terms of shock capturing properties.

The aim of the work described here is to develop a pressure-correction method which is as good as any contemporary, compressible flow technique at shock capturing and at low Mach number degenerating to the incompressible pressure-correction method. To quantify the shock capturing performance of the method, the shocked flow in a quasi-one-dimensional nozzle is calculated using the new pressure-correction method and compared to results from the well known Jameson and MacCormack methods. To move closer to the ultimate ground-effect problem and to show that the pressure-correction shock capturing properties are not degraded in multidimensional problems, a turbulent, under-expanded, axisymmetric, impinging jet calculation is also presented.

Compressible Flow Pressure-Correction Methods

The origins of most pressure-correction methods lie in the Semi-Implicit Method for Pressure Linked Equations (SIMPLE) scheme of Patankar and Spalding,[1] a thorough description of which is given in the book by Patankar.[2] In its basic form, this algorithm is applicable to incompressible, constant density flows and uses pressure and velocity as the main dependent variables. The treatment of the coupling between pressure and velocity is effected by interpreting the mass conservation equation as a constraint equation for the pressure leading to a "guess-and-correct" algorithm for establishing the pressure field.

Issa and Lockwood[3] and more recently Van Doormaal et al.[4] have shown how the scheme may be extended to compressible flows. The idea of continuity as a constraint equation is retained but in modified form. The correct pressure field is that which results in velocities from the momentum equations and density from the equation of state, which together satisfy mass conservation. The implementation suggested by these authors involves a linearization of the mass flux in terms of density and velocity *separately*; further the density appearing in the continuity equation is upwinded. These practices seem to have quite serious effects on the shock capturing properties of existing pressure-correction algorithms: shocks are highly smeared. Rhie[5] and Rhie and Stowers[6] have recently presented pressure-correction schemes that are based on the above idea of mass flux linearization in terms of density and velocity. Shock smearing seems to have been reduced in these algorithms by using an adaptive artifi-

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*Senior Lecturer, Dept. of Mechanical Engineering; currently at Dept. of Transport Technology, Loughborough University, Leicestershire, UK.

†Research Assistant, Dept. of Mechanical Engineering. Member AIAA.

cial viscosity which is less severe than standard upwinding; unfortunately, how this is applied to the mass conservation equation is not made clear. In what follows, the ideas underlying the above algorithms will be referred to as the standard compressible pressure-correction approach.

The first major difference between the schemes above and that presented here is the use of momentum per unit volume as the subject of the momentum equations, rather than velocity (which is momentum per unit mass). This has two major advantages. The time-dependent equations give directly the change in a property per unit volume; whereas the standard pressure-correction schemes must divide the time change by density in order to calculate the new velocity. More importantly, the change in momentum can be reinterpreted as a change in mass flux. This gives a linkage between pressure and mass flux; the mass conservation equation then only contains density in the time-dependent term. It is believed that this fundamental change to previous pressure-correction schemes has great benefits for transonic flow problems.

Quasi-One-Dimensional Flow

The essential details of the new algorithm, and its shock capturing properties, are first demonstrated in a simple one-dimensional test problem.

Governing Equations

The unsteady inviscid flow of a fluid through a quasi-one-dimensional nozzle with area distribution $A(x)$ is defined by

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho u A) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho u A) + \frac{\partial}{\partial x}(\rho u A . u + P . A) = P\frac{\partial A}{\partial x} \tag{2}$$

For a true time-dependent flow calculation, an energy equation must be solved; to simplify the current test calculations the condition of constant total enthalpy H_0 has been assumed. The equation of state can then be expressed as

$$P = \frac{(\gamma - 1)}{\gamma}\rho(H_0 - \tfrac{1}{2}u^2) \tag{3}$$

Discretization Using the New Pressure-Correction Method

Figure 1 shows the staggered grid system and notation used. There are many practical advantages to using a staggered grid; one of the more important is the elimination of "odd-even" pressure decoupling. The inclusion of area terms in variables is a convenience for the quasi-one-dimensional problem and does not invalidate the following discretization schemes. This practice is not used in multidimensional problems.

Integrating the momentum equation over a volume between i and $i + 1$ and time over Δt using the backward Euler implicit scheme gives

$$\begin{aligned} &1/\Delta t[(\rho uA)_{i+1/2} - (\rho uA)^0_{i+1/2}] \\ &+ 1/\Delta x[\tilde{u}_{i+1}(\overrightarrow{\rho u A})_{i+1} - \tilde{u}_i(\overrightarrow{\rho u A})_i] \\ &+ \frac{(P_{i+1} - P_i)}{\Delta x}\cdot\frac{(A_{i+1} + A_i)}{2} = 0 \end{aligned} \tag{4}$$

The sidewall pressure force has been amalgamated with the pressure gradient to give the last term. The superscript 0 denotes the old time level; no superscript denotes the new time value (or the latest estimate of it). The symbol $\tilde{u}$ represents the convecting velocity at the volume face and is interpolated from

$$\tilde{u}_i = 1/2\frac{[(\rho uA)_{i+1/2} + (\rho uA)_{i-1/2}]}{(\rho A)_i} \tag{5}$$

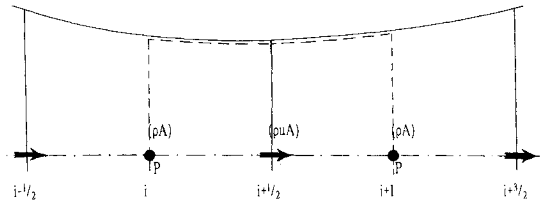

Fig. 1 The staggered grid system.

Also, $(\overrightarrow{\rho u A})$ represents the convected momentum; if flow is assumed from left to right, then $(\overrightarrow{\rho u A})_i$ and $(\overrightarrow{\rho u A})_{i+1}$ in Eq. (4) can be written in terms of their values at $i \pm 1/2$ using the upwind formula

$$(\overrightarrow{\rho u A})_i = (\rho uA)_{i-1/2} \tag{6}$$

[Note: this is exactly opposite to the practice in the standard method where it is considered that the mass flux convects the velocity and so (ρuA) would be interpolated and u upwinded].

The discretized momentum equation is then in the form

$$D\,(\rho uA)_{i-1/2} + E\,(\rho uA)_{i+1/2} + F\,(\rho uA)_{i+3/2} = Q_m \tag{7}$$

where

$$D = -\tilde{u}_i$$

$$E = \frac{\Delta x}{\Delta t} + \tilde{u}_{i+1}$$

$$F = 0$$

$$Q_m = \frac{\Delta x}{\Delta t}(\rho uA)^0_{i+1/2} - (P_{i+1} - P_i)\frac{(A_{i+1} + A_i)}{2}$$

This is implicitly solved using the pressure field at the end of the previous time step to give a new estimate of the momentum field at the new time level.

Similarly, the continuity equation is integrated over time and the volume between $i - 1/2$ and $i + 1/2$

$$1/\Delta t[(\rho A)_i - (\rho A)^0_i] + 1/\Delta x[(\rho uA)_{i+1/2} - (\rho uA)_{i-1/2}] = Q_d \tag{8}$$

With the provisional values of momentum and density (unless iterating the density is taken as the value at the beginning of the time step), Q_d will not be zero. The pressure field is adjusted producing a change δ in the density and momentum fields such that Q_d is reduced to zero

$$1/\Delta t[\delta(\rho A)_i] + 1/\Delta x[\delta(\rho uA)_{i+1/2} - \delta(\rho uA)_{i-1/2}] = -Q_d \tag{9}$$

The effect of the pressure field adjustment on the momentum field can be found from the discretized momentum Eq. (7). In δ form this is

$$\begin{aligned} &D\,\delta(\rho uA)_{i-1/2} + E\,\delta(\rho uA)_{i+1/2} \\ &= -(\delta P_{i+1} - \delta P_i)\frac{(A_{i+1} + A_i)}{2} \end{aligned} \tag{10}$$

The standard SIMPLE approximation is now introduced: since coefficient D is smaller than coefficient E (or can be made so by a suitable choice of time step), then the main influence of the change in pressure gradient is assumed to be upon the central $i + 1/2$ cell giving the approximate equation

$$\begin{aligned} \delta(\rho uA)_{i+1/2} &= -1/2\frac{(A_{i+1} + A_i)}{\left(\dfrac{\Delta x}{\Delta t} + \tilde{u}_{i+1}\right)}(\delta P_{i+1} - \delta P_i) \\ &= S_{pm_{i+1/2}}(\delta P_{i+1} - \delta P_i) \end{aligned} \tag{11}$$

Since this is a corrector equation, any errors introduced by this approximation will be absent in the steady-state solution. The linkage between density and pressure is found from an equation of state,

$$\delta(\rho A)_i = A_i\,\delta P_i \frac{(\gamma/(\gamma-1))}{(H_0 - 1/2\tilde{u}_i^2)} = S_{pd_i}\,\delta P_i \tag{12}$$

[Note again, in the standard method, change in pressure is related to velocity, and so, to make the equation tractable a further approximation is introduced by linearizing the term $\delta(\rho u A)$ as $Au\,\delta\rho + A\rho\,\delta u$.]

The mass conservation equation can thus be reduced to an equation for the pressure correction field

$$D\,\delta P_{i-1} + E\,\delta P_i + F\,\delta P_{i+1} = -Q_d \tag{13}$$

where

$$D = S_{pm_{i-1/2}}$$

$$E = \frac{\Delta x}{\Delta t} S_{pd_i} - S_{pm_{i-1/2}} - S_{pm_{i+1/2}}$$

$$F = S_{pm_{i+1/2}}$$

This equation is implicitly solved for the change in pressure, which is then used to update the momentum and density fields via Eqs. (11) and (12). This sequence can be iterated each time step, although in general the time step is small, and iteration is only needed when the flow is changing rapidly.

This scheme is physically incorrect for supersonic flows since the pressure-momentum linkage does not model the hyperbolic behavior of the system of equations; at all flow speeds, the pressure-correction equation remains essentially elliptic. To remedy this defect, the hyperbolic nature of the flow was simulated by the use of a "retarded pressure." Wornom and Hafez[7] first introduced the concept for a similar flow problem, but it is also closely related to ideas developed for transonic potential flow such as the artificial density of Hafez et al.[8] and the artificial viscosity of Jameson.[9]

The retarded pressure field is defined by

$$\bar{P} = P + \mu\,\overleftarrow{\Delta x P_x} \tag{14}$$

where the arrow represents that the derivative is biased in the upstream direction and μ is a function to be defined. So in discretized form for flow from left to right

$$\bar{P}_i = P_i + \mu_{i-1/2}(P_i - P_{i-1}) \tag{15}$$

The function μ is defined by

$$\mu_{i-1/2} = \max\left\{0, k\left[1 - (M_{\text{ref}}/M_{i-1/2})^2\right]\right\} \tag{16}$$

where $M_{i-1/2}$ is the upstream face Mach number and k and M_{ref} are constants of the order unity. The retarded pressure field is substituted for the real pressure in the momentum and pressure-correction equations. It should be noted that the change in density needs to be linked via the equation of state to the change in real pressure; this can only be accomplished in the pressure-correction equation if real pressure is an explicit function of retarded pressure. Consequently, the real to retarded transformation must be implicit

$$\bar{P}_i = P_i + \mu_{i-1/2}(\bar{P}_i - \bar{P}_{i-1}) \tag{17}$$

and the inverse transformation is

$$P_i = \bar{P}_i - \mu_{i-1/2}(\bar{P}_i - \bar{P}_{i-1}) \tag{18}$$

To summarize, pressure is transformed to retarded pressure using Eq. (17). The momentum and pressure correction equations are solved yielding a new momentum and retarded pressure field. The retarded pressure is then transformed back to the real pressure using Eq. (17) in order to determine the density field. A discussion of the form and importance of the retarded pressure function μ is given later.

Program Implementations

The new pressure-correction method was initially implemented specifically for the quasi-one-dimensional problem. The well known Jameson[10,11] and MacCormack[12,13] explicit schemes were coded in similar manner for this test problem. A common library of subroutines was used for input, output, and general utility functions. The same coding style removes any timing bias due to individual programming skill. Also, the tests were executed on a personal microcomputer giving a high degree of repeatability as time sharing, virtual memory, or other large operating system overheads were eliminated.

The Jameson program is nonmultigrid, three stage-time stepping but with no residual averaging. The artificial dissipation is a mixture of second and fourth difference terms as specified in the work of Jameson.[10]

The MacCormack program is based on the 1969 explicit predictor-corrector method. The direction of the predictor step is alternated in successive time steps. Explicit addition of artificial dissipation was found to be necessary to suppress oscillations near the shock, and so the Jameson second-difference terms were used.

Clearly, improvements have been made in both these methods, but they are chosen here as simple, standard techniques representing algorithms accepted as having adequate shock capturing properties.

To show how poorly the standard, compressible, pressure-correction method captures shock waves, an old code was adapted for this problem; results from this code are included here merely to show the enhanced ability of the new scheme, and they do not form part of the formal comparison.

Test Problem

The nozzle configuration used in this comparison is that devised by Denton[14] and adopted in the calculations of Moore and Moore.[15] The distribution of area is designed so that, for shock free flow, the Mach number increases linearly with distance according to the formula

$$x = 10.0 + 45.0 \times (M - 1) \tag{19}$$

In the present comparisons, the nozzle profile is defined for an axial distance x of one unit to 46 units with a throat of unity area at 10 units. The ratio of outlet to inlet area is 1.39. The computational grid has uniform spacing of one unit.

Initial conditions for all variables were set over the whole domain as being constant and equal to the known analytical solution at the inlet with the exception of the exit pressure, which was set at a fixed value. The inlet boundary condition consisted of extrapolating the static pressure from the interior, which together with given values of total pressure and total temperature allows an isentropic calculation of the other variables. At the outlet the static pressure was held fixed, and any unknown variables found from extrapolation.

Each program has a set of parameters that affect its behavior by controlling the physical time step of the calculation and the amount of dissipation present. Experimentation allowed a reasonable, representative (though probably not optimal) set of parameters for the three programs; no further changes were allowed throughout the course of the comparison. Details of these parameters are given in Table 1. For the MacCormack and Jameson programs, the time step is very important in controlling the rate of convergence, this was set close to the maximum allowable for stability. The pressure-correction program was stable for much larger time steps; however, the

Table 1 Program parameters

	Jameson	MacCormack	Pressure correction
Dissipation[a] $k^{(2)}$	1.5	1.5	—
$k^{(4)}$	0.1	0.0	—
Time step (s)	2.5×10^{-3}	1.5×10^{-3}	6.0×10^{-3}

[a]The $k^{(2)}$ and $k^{(4)}$ as defined by Jameson.[10]

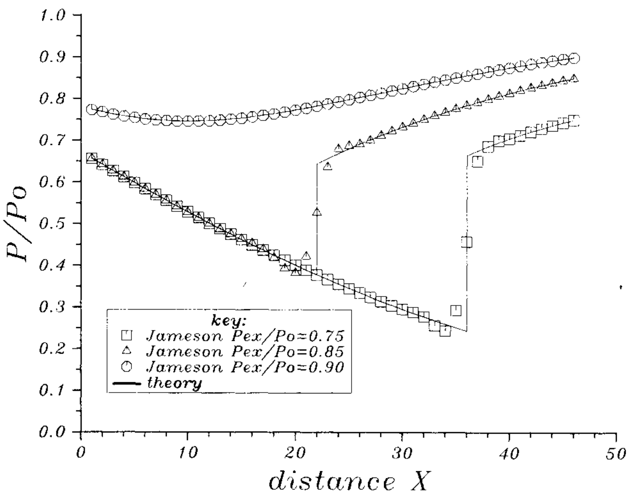

Fig. 2a Pressure distribution for Jameson method.

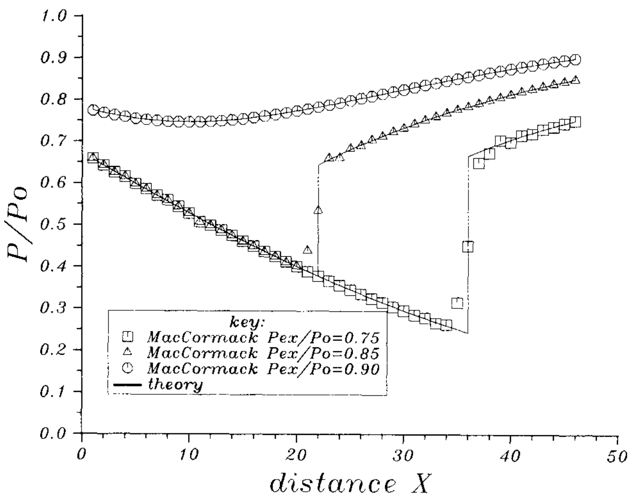

Fig. 2b Pressure distribution for MacCormack method.

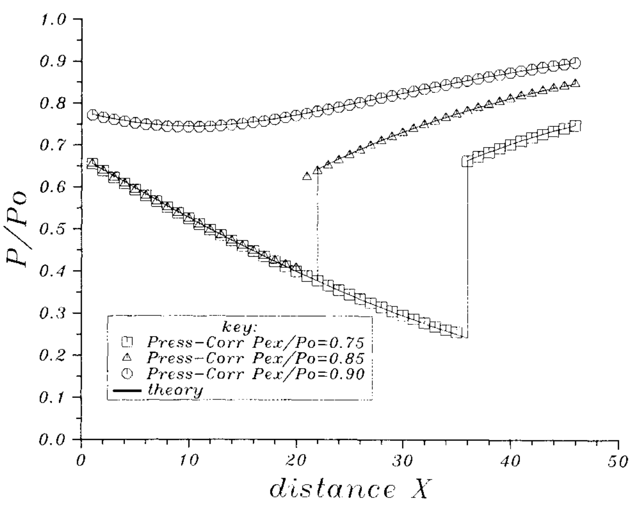

Fig. 2c Pressure distribution for new pressure-correction method.

chosen value is a compromise to give reasonable accuracy without needing to iterate each time step.

The first test case was for a ratio of exit static pressure to inlet total pressure of 0.75 giving a fairly strong shock with a peak Mach number of 1.58. Convergence of the calculation in the steady state was judged by monitoring the root mean square value of the rate of change of density with time—since the time step was of the order of 10^{-3} s, a rate of change smaller than 10^{-3} gives a change in density that is close to the single precision machine accuracy. For the second case, the initial conditions were taken as the converged strong shock solution and the exit pressure ratio was increased to 0.85. This causes the shock to move towards the throat and weaken with a peak upstream Mach number of 1.27. The criterion for convergence to the steady state was as in the previous case. Finally, with the weak shock solution as the initial conditions, the exit pressure was raised to give a ratio of 0.90 so that the flow became totally subsonic.

Results

Shown in Figs. 2a–2c are the steady-state pressure distributions, normalized by the inlet total pressure, for the Jameson, MacCormack, and new pressure-correction methods. Table 2 gives details of the achieved measure of convergence, number of time steps, and processing time for the three programs. For the purposes of this work, the most important features of the results are the sharpness of shock capturing, accuracy of shock positioning, and speed of computation.

The dissipation common to both MacCormack and Jameson programs gives similar shock smearing—in both cases over 4–6 grid nodes. Undershoots adjacent to the shock are less apparent in the MacCormack results, probably because the predictor-corrector scheme has an inherent dissipation that is absent in the Jameson method. The pressure correction program captures both shocks very sharply over two nodes, with no under- or overshoots. A very slight amount of smearing is apparent in that the low pressure node is fractionally too high in both cases.

Shock positioning is good for all programs. The only concern is the position of the weak shock for the pressure-correction method, which is displaced, but only by one node, upstream. At this location the nozzle is almost parallel, and so the shock positioning is very sensitive to the downstream pressure. It is really only the sharpness of the shock capturing that highlights this small error.

Examination of Table 2 shows that the pressure-correction method is the most economic in terms of processing time to reach the steady state—in fact it is around three times

Table 2 Details of computing time and convergence

Strong shock	Jameson	MacCormack	Pressure correction
No. of time steps	900	1600	500
C.P.U. time[a] (s)	654	843	230
Convergence[b]	−3.5	−3.6	−3.6
Weak shock			
No. of time steps	1350	1700	650
C.P.U. time[a] (s)	986	897	315
Convergence[b]	−3.3	−3.1	−3.1
Subsonic			
No. of time steps	1500	2900	850
C.P.U. time[a] (s)	1092	1531	356
Convergence[b]	−3.0	−3.0	−3.1

[a]On Atari 1040ST microcomputer.
[b]Defined as $\log_{10}\{\mathrm{RMS}(\delta\rho/\delta t)\}$.

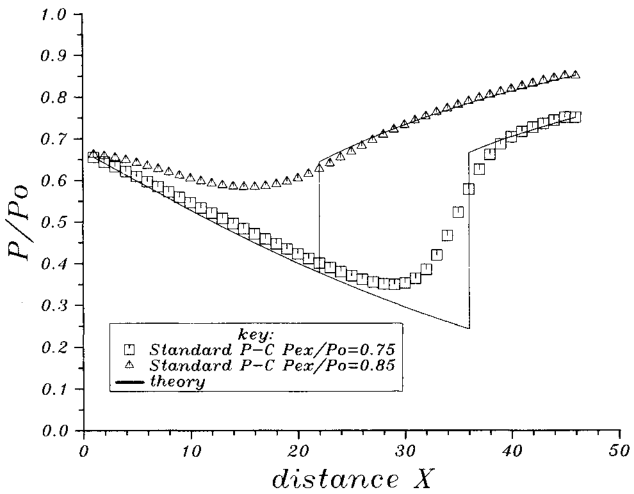

Fig. 3 Shock smearing by standard, pressure-correction method.

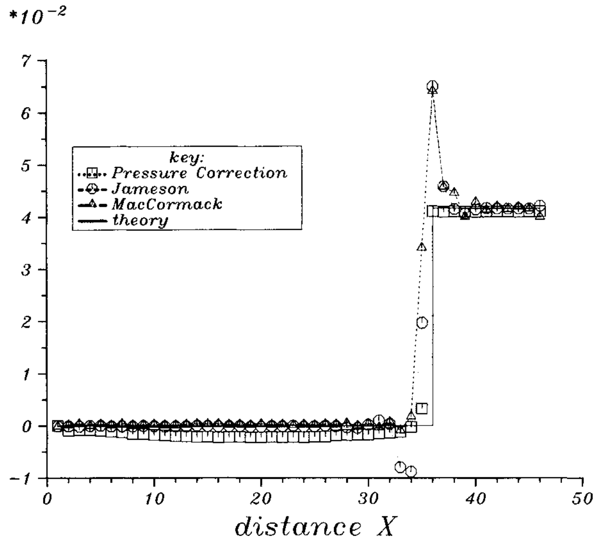

Fig. 4 Entropy distributions Σ.

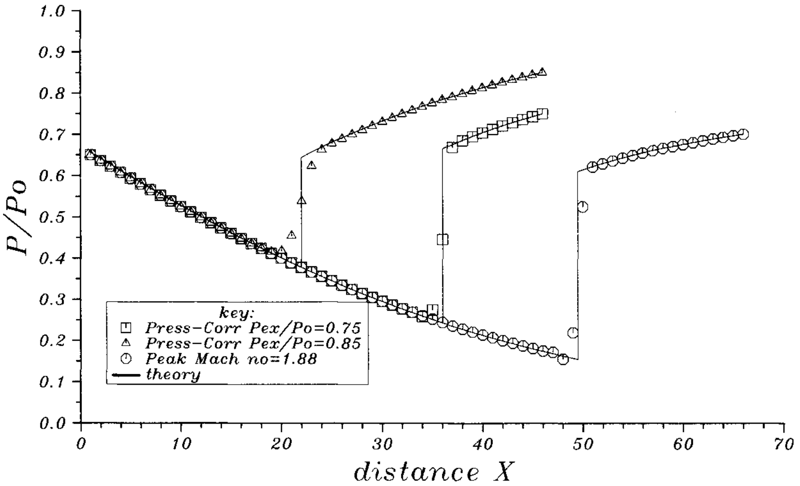

Fig. 5 Higher Mach number shock capturing.

quicker than the other schemes for these cases. This advantage is due to the larger time step; all the methods take a similar processing time per time step. It should be noted that the Jameson and MacCormack schemes employ "second-order," accurate time-stepping algorithms; whereas the pressure-correction time-stepping is only "first-order" accurate. Obviously with a larger time step, the transient accuracy is reduced, meaning that a longer physical time is required to reach the steady state.

Figure 3 shows the weak and strong shock case for the standard compressible pressure correction method; the shock capturing performance is completely unacceptable; indeed the weak shock becomes smeared into the throat causing itself to disappear and the nozzle to unchoke.

A more critical analysis of the results can be made by examining the entropy distribution—a good measure of entropy is the term Σ, defined by

$$\Sigma = (P/P_\infty)/(\rho/\rho_\infty)^{\gamma-1} \tag{20}$$

The subscript ∞ denotes the theoretical inlet values. This should be constant for any part of the flow that is shock free and should increase discontinuously through a shock wave. The distributions of Σ are shown in Fig. 4 for the strong shock case. A small error is noticeable up to the throat region of the pressure-correction results; this is possibly due to the retarded pressure transformation. Again no over- or undershoots can be discerned around the shock for the pressure-correction results, and the subsonic branch is much more accurate than the other two methods—this is a vindication of using pressure-correction-based methods for subsonic zones.

Overall the pressure-correction method performs surprisingly well, its shock capturing is very sharp, and it is considerably faster than the tested versions of the widely used Jameson and MacCormack schemes.

Choice of the Retarded Pressure Function

When developing the new pressure-correction method, the function for μ was as in Eq. (16) with $k = 1$ and $M_{\text{ref}} = 1$. This gave good shock capturing but under certain conditions, an unphysical expansion shock was created at the throat. Wornom,[16] using a different calculation method but with the retarded pressure concept, encountered a similar problem. He set $M_{\text{ref}} = 0.9$ to give dissipation before the sonic point but still has small errors near the throat.

We have taken a slightly different approach: a second term is introduced to the retarded pressure function to become

$$\mu_{i-\frac{1}{2}} = \max\Big\{0, k1\big[1-(M_{\text{ref}1}/M_{i-\frac{1}{2}})^2\big] \quad k2\big[1-(M_{\text{ref}2}/M_{i-\frac{1}{2}})^2\big]\Big\} \tag{21}$$

The first term still has unity constants and the second term $k2 = 0.7$ and $M_{\mathrm{ref2}} = 0.7$, but, if the pressure is increasing in the flow direction, $k2$ is locally set to zero. Thus, the retarded pressure function switches on before a sonic point, reverts to the unity function as the Mach number increases, and switches off through a shock wave. This function was used in the calculations previously shown. It is important to realize that the retarded pressure is used to add dissipation to stabilize the supersonic part of the flow calculation *not* to help shock capturing; indeed its presence in a shock wave will cause smearing.

For calculations with shock waves much stronger than those previously presented, it was found that the sharp pressure rise was likely to induce an instability in the calculation at the foot of the shock. For a more robust calculation at Mach numbers above around 1.6, it is useful to have some dissipation through the shock wave. For example Fig. 5 shows results for a single-term, retarded pressure function with a $k1 = 0.5$ and $M_{\mathrm{ref1}} = 0.775$. The Denton nozzle has been extended to 66 units in order to allow calculations with a higher Mach number. The strongest shock wave shown has a theoretical peak upstream Mach number of 1.88. For comparison the two shock cases are shown with the new retarded pressure function. The shock capturing performance is still very good but not as exceptional as the previous results. A slight undershoot is discernible for the very strong shock case.

Thus, the choice of retarded pressure function is critical in determining the performance of the new method. Other forms for the function may give very sharp shock capturing at Mach numbers higher than those presented, but more work is required to throw light on the high Mach number capabilities of the method.

Axisymmetric Impinging Jet Problem

A more realistic test of the new pressure-correction method is the calculation of an axisymmetric impinging jet.

A new program was written based on the quasi-one-dimensional pressure-correction program for axisymmetric and plane two-dimensional problems. This solves the Reynolds-averaged, time-dependent, Navier-Stokes equations using the eddy viscosity $k - \varepsilon$ turbulence model. The coefficients of this model are the standard set, e.g., Launder and Spalding.[17] Wall functions are used at the solid boundary to avoid the need for excessive grid clustering at walls.

The finite-volume conservation equations are discretized and solved sequentially by a line Gauss-Seidel technique and applied alternately in each of the coordinate directions. The program essentially embodies the methods described by Patankar[2] when cast in terms of properties per unit volume. For the present flow problem, the retarded pressure concept is simply extended by considering the interface Mach numbers based on the convecting component velocities (M_x and M_y) and transforming by using pressure gradients in both directions. Impinging jet flows are such that any high Mach number flow is in the positive coordinate directions, and thus the transformation becomes

$$\bar{P}_{ij} = P_{ij} + \mu_{xi-1/2j}(\bar{P}_{ij} - \bar{P}_{i-1j}) + \mu_{yij-1/2}(\bar{P}_{ij} - \bar{P}_{ij-1}) \qquad (22)$$

The retarded pressure functions μ_x and μ_y are calculated from Eq. (21) using the corresponding component Mach numbers M_x and M_y.

The test problem chosen corresponds to the Donaldson and Snedeker[18] moderately under-expanded impinging jet. The convergent jet nozzle is approximately 2 diam from the impingement plane. The reservoir total pressure is 2.69 times the ambient pressure giving a static pressure at the nozzle exit 40% greater than ambient.

Figure 6 shows the configuration of the problem and a streakline plot of the steady-state flow; the stars represent the initial positions of the particles. The widely varying lengths of particle track displays the large range of velocity scales present. There are four different boundary conditions used in this calculation. The jet is created by extrapolating pressure from the interior and using conditions of constant total pressure and total temperature to calculate the density and mass flux; the flow direction is specified. The centerline of the jet is treated as a symmetry plane and the impingement plane as a no-slip wall. The other boundaries are "floating" constant pressure boundaries, the pressure on these boundaries is allowed to adjust as a fixed proportion of the interior and the

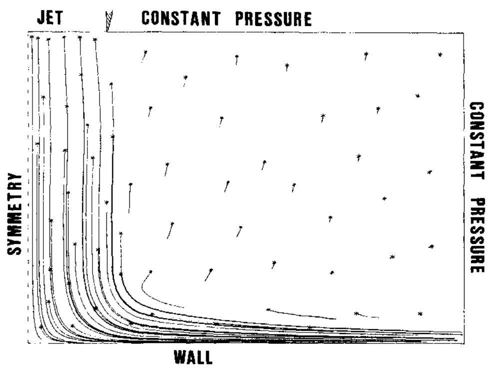

Fig. 6 Impinging jet configuration streaklines.

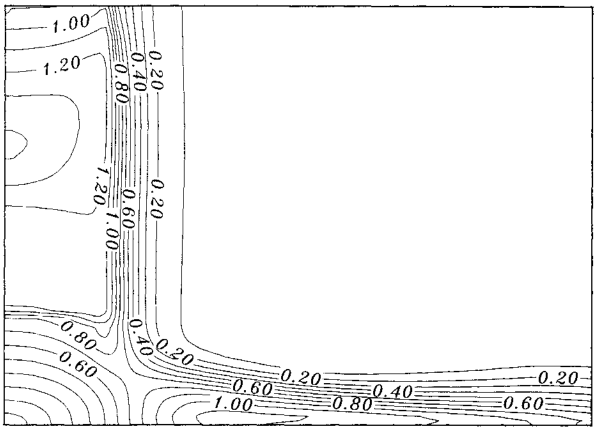

Fig. 7 Mach number contours.

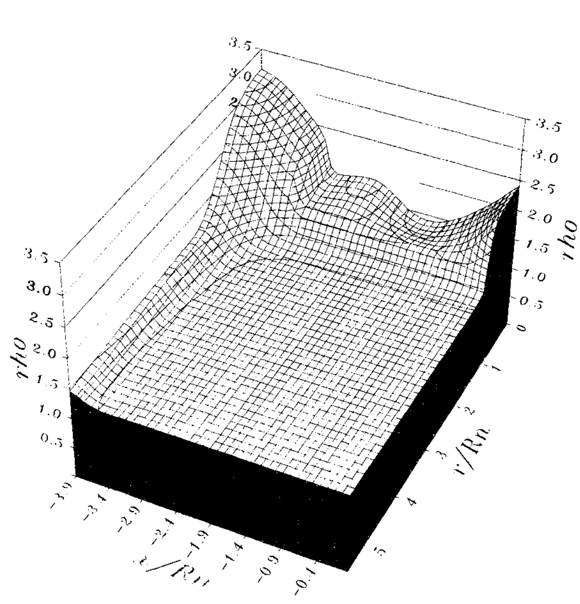

Fig. 8 Density surface plot.

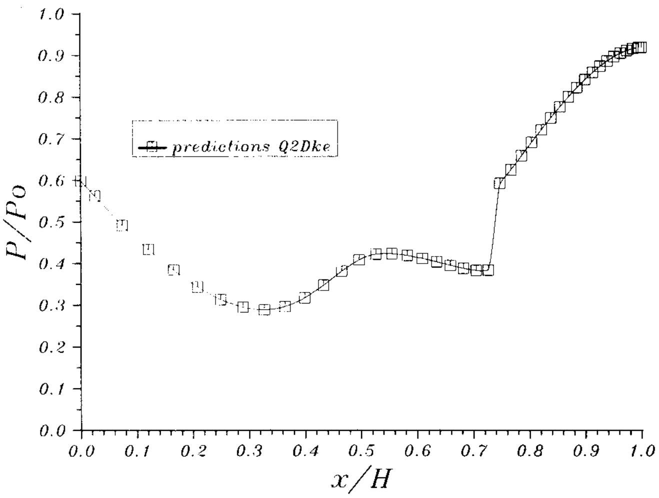

Fig. 9 Jet centerline pressure distribution.

given "atmospheric" pressure; fluid can be entrained and expelled through these boundaries with no *a priori* assumptions of flow direction.

Contour plots for Mach number and density on a 40 × 40 cell mesh are shown in Figs. 7 and 8. The density plot clearly shows the flow under expanded at the nozzle lip followed by a periodic overexpansion and under expansion before the structure is terminated by a strong standoff shock. Experimental work by Love et al.[19] shows, that for this jet pressure ratio, the wavelength of the structure should be 1 nozzle diameter, and the calculations reproduce this. The peak Mach number in this case is 1.4. Interestingly, the Mach number profiles indicate that a significant supersonic zone exists in the wall jet at a location under the edge of the jet.

Figure 9 is the calculated pressure distribution, normalized by the jet total pressure, along the axial centerline of the jet. It can be seen that, due to grid clustering near the impingement plane, there are relatively few computational nodes in the core of the jet, but the expansion and compression processes are still resolved, and the shock is captured sharply. Note the difference between the rise in pressure across the shock wave and that in the turbulent subsonic impingement zone. If the flow is aligned along one of the grid directions, then shock capturing should be as good as in the one-dimensional examples.

This last test case demonstrates clearly that the new pressure-correction method can accurately model a flow with supersonic regions, shock waves, and almost stagnant turbulent mixing. It therefore forms a suitable starting point for ASTOVL ground-effect problems.

Conclusions

This paper has briefly described the methodology of a new pressure-correction algorithm, which is suitable for the calculation of transonic flows which also contain turbulent, low Mach number zones. The results presented for a quasi-one-dimensional flow problem indicate very promising performance of the present algorithm in terms of shock capturing and speed of calculation; the method, in fact, out performs the explicit MacCormack and Jameson schemes used as a comparison for this case. Initial computations of a turbulent, under-expanded, impinging jet flow show that the scheme is practical and accurate for realistic problems. Further work is required to explore the behavior at Mach numbers greater than about 1.9. Nevertheless, the scheme has been shown to work successfully over the Mach number range likely to be of interest in future ASTOVL problems.

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References

[1]Patankar, S. V., and Spalding, D. B., "A Calculation Procedure for Heat, Mass and Momentum Transfer in Three-Dimensional Parabolic Flows," *International Journal of Heat and Mass Transfer*, Vol. 15, 1972, p. 1787.

[2]Patankar, S. V., *Numerical Heat Transfer and Fluid Flow*, McGraw-Hill, Washington, DC, 1980.

[3]Issa, R. I., and Lockwood, F. C., "On the Prediction of Two-Dimensional Supersonic Viscous Interactions Near Walls," *AIAA Journal*, Vol. 15, No. 2, Feb. 1977, pp. 182–188.

[4]Van Doormaal, J. P., Raithby, G. D., and McDonald, B. H., "The Segregated Approach to Predicting Viscous Compressible Fluid Flows," *ASME Journal of Turbo-machinery*, Vol. 109, April 1987, pp. 268–277.

[5]Rhie, C. M., "A Pressure Based Navier-Stokes Solver Using the Multigrid Method," AIAA Paper 86-0207, Jan. 1986.

[6]Rhie, C. M., and Stowers, S. T., "Navier-Stokes Analysis for High Speed Flows Using a Pressure Correction Algorithm," AIAA Paper 87-1980, June 1987.

[7]Wornom, S. F., and Hafez, M. M., "Calculations of Quasi-One Dimensional Flows with Shocks," *Computers & Fluids*, Vol. 14, No. 2, 1986, pp. 131–140.

[8]Hafez, M., South, J., and Murman, E., "Artificial Compressibility Methods for Numerical Solutions of Transonic Full Potential Equation," *AIAA Journal*, Vol. 17, No. 8, 1979, pp. 838–844.

[9]Jameson, A., "Transonic Potential Flow Calculations in Conservation Form," *Proceedings of the 2nd Computational Fluid Dynamics Conference*, Hartford, 1975, pp. 148–161.

[10]Jameson, A., "Transonic Aerofoil Calculations using the Euler Equations," *Numerical Methods in Aeronautical Fluid Dynamics*, edited by P. L. Roe, Academic Press, London, 1982, pp. 289–308.

[11]Jameson, A., "A Vertex Based Multigrid Algorithm for Three Dimensional Compressible Flow Calculations," *Winter Annual Meeting of ASME*, Anaheim, CA, Dec. 1986, pp. 45–73.

[12]MacCormack, R. W., "The Effect of Viscosity on Hypervelocity Impact Cratering," AIAA Paper 69-354, 1969.

[13]MacCormack, R. W., "A Numerical Method for Solving the Equations of Compressible Viscous Flow," *AIAA Journal*, Vol. 20, No. 9, Sept. 1982, pp. 1275–1281.

[14]Denton, J. D., "An Improved Time Marching Method for Turbomachinery Flow Calculation," ASME, *Journal of Engineering for Power*, Vol. 105, No. 3, July 1983, pp. 514-521.

[15]Moore, J. G., and Moore, J., "Shock Capturing Using Pressure Interpolation," Virginia Polytechnic Institute and State University, Turbomachinery Research Group Rpt. No. JM/86-1, 1986.

[16]Wornom, S. F., "A Two-Point Difference Scheme for Computing Steady-State Solutions to the Conservative One-Dimensional Euler Equations," *Computers & Fluids*, Vol. 12, No. 1, 1984, pp. 11–30.

[17]Launder, B. E., and Spalding, D. B., "The Numerical Computation of Turbulent Flows," *Computer Methods in Applied Mechanics and Engineering*, Vol. 3, No. 2, 1974, pp. 269–289.

[18]Donaldson, C. D., and Snedeker, R. S., "A Study of Free Jet Impingement. Part I. Mean Properties of Free and Impinging Jets," *Journal of Fluid Mechanics*, Vol. 45, Part 2, 1971, pp. 281–319.

[19]Love, E. S., Grigsby, C. E., Lee, L. P., and Woodling, J. M., "Experimental and Theoretical Studies of Axisymmetric Free Jets," NASA TR-R-6, 1959.